AN ODE TO GEOMETRY

Workshop 16–21 September 2024

Departamento de Matemática, Instituto Superior Técnico
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List of Abstracts

Sílvia Anjos (IST, Lisboa, PT)

Title: *The space of symplectic balls in rational 4-manifolds*

Abstract: Existence of symplectic embeddings of k disjoint balls of given capacity $c_1, \ldots, c_k$ into a given symplectic manifold (M, ω) is a central problem in symplectic topology. However, besides a few examples, very little is known about the space of all such embeddings. In this talk, I will discuss the case of rational 4-manifolds with small Euler numbers, with a special attention to the case of the projective plane. In this case, our work reveals an interesting connection with the complex geometry of the configuration space of k distinct points in $\mathbb{CP}^2$. This is based on joint works with Jarek Kedra, Tian-Jun Li, Jun Li and Martin Pinsonnault.

José Julián Barragán Amado (University of Lisbon, Lisboa, PT)

Title: *The effect of resummation on retarded Green's function and greybody factor in AdS black holes*

Abstract: In this talk, we introduce the retarded Green's function and the greybody factor in asymptotically AdS black holes. Using the connection coefficients of the Heun equation, expressed in terms of the Nekrasov-Shatashvili (NS) free energy of an $SU(2)$ supersymmetric gauge theory with four fundamental hypermultiplets, we derive asymptotic expansions for both the retarded Green's function and the greybody factor in the small horizon limit. Furthermore, we compute the corrections to these asymptotic expansions resulting from the resummation procedure of the instanton part of the NS function. This presentation is based on joint work with Shankhadeep Chakrabortty and Arpit Maurya.

Bruno Carneiro da Cunha (UFPE, Recife, Pernambuco, Brasil)

Title: *Isomonodromy and the accessory parameter problem for conformal maps*

Abstract: A classical result in complex analysis relates the uniformizing map of a given simple domain to the ratio of solutions of a second order ordinary differential equation of the Fuchsian type. Despite its simplicity, the usefulness of this relation to the actual construction of the map has been

hindered by the problem of fixing the parameters of the equation given the geometrical information of the domain. In this talk we will outline a solution for this problem for simply-connected domains bounded by arcs of circles, as well as present the new type of special functions involved in the solution, the semi-classical conformal block. We will illustrate the technique by considering a few examples numerically.

Ana Bela Cruzeiro (IST, Lisboa, PT)

Title: *Geometric and variational aspects of the Euler and the Navier–Stokes equations*

Abstract: We present symmetry reduction for stochastic Lagrangian systems with advected quantities whose configuration space is a Lie group. Such variational principles yield deterministic constrained variational principles for dissipative equations of motion. The general theory is presented for the finite dimensional situation. In infinite dimensions we obtain partial differential equations. We apply this technique to the compressible Navier-Stokes equation, generalizing Arnold's description of the Euler equation. This is joint work with Xin Chen and Tudor Ratiu.

Gabriele Degano (University of Lisbon, Lisboa, PT)

Title: *Q and T functions for the ground state of Quantum KdV in the semiclassical limit*

Abstract: In this talk, we describe an anharmonic oscillator known as the ground state potential for the quantum KdV model. We study the spectral determinant associated with the central connection problem and analyze its zeros in various asymptotic regimes as the degree grows large. We also provide explicit formulas for the leading-order behaviors.

Jean Douçot (University of Lisbon, Lisboa, PT)

Title: *Dualities between genus zero wild nonabelian Hodge spaces*

Abstract: When allowing for irregular singularities, to get a nice moduli space of meromorphic connections, one has to choose as initial datum a wild Riemann surface, which encodes the formal normal form of the connections at their singularities. The intriguing observation that moduli spaces corresponding to connections with different ranks, number of singularities, and pole orders, may be isomorphic, and the related fact that many Painlevé-type équations admit several different Lax representations, raises the question of understanding more systematically such dualities. In this talk, we will describe a combinatorial way of obtaining, from the datum of a fission tree of generic form, different genus zero wild Riemann surfaces which are expected to give rise to isomorphic non-abelian Hodge spaces. This partially generalizes to the case of arbitrary singularity data the picture of the so-called simply-laced case due to P. Boalch, where several equivalent isomonodromy systems can be reconstructed from the datum of a supernova quiver.

Pavlo Gavrylenko (SISSA, Trieste, IT)

Title: *Isomonodromic deformations on a torus, Fredholm determinant, and connection constant*

Abstract: We will see how isomonodromic tau functions on a torus with one puncture can be expressed as a matrix Fredholm determinant. This Fredholm determinant can be used to prove Kyiv formula in the toric case. We also use it to study the monodromy dependence of the tau

functions, and then to find the connection constant for the modular transformation of these tau functions. This connection constant can also be easily turned into the fusion kernel for c=1 toric conformal blocks. The talk will be based on a joint work with Fabrizio Del Monte and Harini Desiraju.

Davide Guzzetti (SISSA, Trieste, IT)

Title: *Uniqueness of Asymptotic solutions for linear ODEs with not necessarily meromorphic coefficients – a Levinson type theorem*

Abstract: We consider systems of linear ODEs with analytic coefficients on sectorial domains, which are asymptotically diagonal for large values of the independent variable z. Under some conditions, we show the existence and uniqueness of an asymptotic fundamental matrix solution on a big sectorial domain. This includes systems depending on parameters. The theorem is of Levinson type, but while Levinson's approach only predicts existence, we also establish uniqueness.The result applies to systems of ODEs with not necessarily meromorphic coefficients, the leading diagonal term of the matrix coefficient being a generalized polynomial in z with real exponents (equations arising ODE/IM correspondence are of this kind). Another application is the classical case of ODEs with meromorphic coefficients at a ramified singularity, for which we provide sufficient conditions to guarantee the uniqueness of a fundamental solution with prescribed asymptotics, and we explicitly characterize the optimal sector. This is a joint work with G. Cotti and D. Masoero (https://arxiv.org/pdf/2310.19739).

Claus Hertling (University of Mannheim, Mannheim, DE)

Title: *Upper triangular matrices and induced structures: vanishing cycles, monodromy groups, distinguished bases, braid group orbits, moduli spaces*

Abstract: Upper triangular matrices with ones on the diagonal and entries which are integers (or algebraic integers) arise in many contexts, e.g. as Stokes matrices in the theory of meromorphic connections with irregular poles, in many situations in algebraic geometry (often related to Stokes matrices), especially in quantum cohomology and the theory of isolated hypersurface singularities, but also in the theory of Coxeter groups. Concepts from singularity theory like vanishing cycles, monodromy groups, Seifert forms, tuples of (pseudo-)reflections and distinguished bases can be derived from upper triangular matrices in cases beyond singularity theory and are interesting to study. Additionally, always braid group actions on the matrices and on the distinguished bases are in the background. They give rise to certain covering spaces of the classifying space of the braid group. These are interesting natural global manifolds. Some are well known, others are new. The talk presents concepts and old and new results. It puts emphasis on some cases from singularity theory and some 3×3 cases.

Andreas Hohl (KU Leuven, Leuven, BE)

Title: *The Fourier transform of Stokes local systems*

Abstract: Stokes data encode information about irregular singularities of differential equations, and in general it is a difficult task to describe how they behave under Fourier transform. Building on a recent work of T. Mochizuki and using the language of Stokes local systems of P. Boalch, one

obtains an explicit and algorithmic such transformation rule in the case of systems of pure level. In this talk, I will explain the concept of a Stokes local system as well as our algorithm, and I will in particular illustrate it in concrete examples. This is joint work with Jean Douçot.

Hiroshi Iritani (Kyoto University, Kyoto, JP)

Title: *Fourier transformation and equivariant quantum cohomology*

Abstract: Equivariant quantum cohomology has the structure of a difference module with respect to the equivariant parameters. The difference module structure is given by the so-called shift (or Seidel) operators, defined by counting holomorphic sections of certain fibre bundles. A conjecture of Teleman says that the quantum cohomology (differential equation) of a GIT quotient should be related to the equivariant quantum cohomology (difference equation) of the prequotient by a Fourier (or Mellin) transformation. I will discuss this Fourier duality through several examples.

Martin Klimes (University of Zagreb, Zagreb, HR)

Title: *Deformations of singularities of meromorphic $\mathfrak{sl}(2,\mathbb{C})$-connections over Riemann surfaces*

Abstract: I will present a general theory of confluences and degenerations for singularities of traceless 2×2 meromorphic connections and their isomonodromic deformations. Namely, I'll explain how Stokes data are attached to an underlying geometry of meromorphic quadratic differentials, and how this works in the confluent setting. A motivation comes from study of the degeneration process of Painlevé monodromy manifolds.

Oleg Lisovyi (université de Tours, Tours, FR)

Title: *Connection problems for Heun and Mathieu equations and quasiclassical conformal blocks*

Abstract: The connection problem for Heun and Mathieu equations seeks to relate canonical bases of solutions associated to different singular points. Recently, Bonelli, Iossa, Lichtig and Tanzini have proposed a conjecture relating the relevant connection matrix to quasiclassical conformal blocks of the Virasoro algebra. In practical terms, their conjecture allows to compute the connection coefficients in the form of a perturbative expansion in a suitable parameter. I will explain how to derive the corresponding perturbative formulas in two different ways without using CFT.

Gonçalo Oliveira (IST, Lisboa, PT)

Title: *Geodesics, Electrostatics and Solids*

Abstract: Based on recent joint work with Christopher Fillmore and Herbert Edelsbrunner I will explain how to beat the world record for the number of closed geodesics of index one on a K3 surface. Such geodesics are related to some conjectures originating in the Physics literature and their construction is related to an open problem in electrostatics posed by Maxwell in 1873.

Andrea Raimondo (Università di Bergamo, Bergamo, IT)

Title: *Affine Gaudin Bethe Ansatz and the ODE/IM correspondence*

Abstract: We consider the monodromy-free opers corresponding to solutions of the Affine Gaudin Bethe Ansatz equations. We define and study the spectral determinants (called Q functions) for these opers. We conjecture that the Q-functions obtained from the Affine Gaudin Bethe Ansatz coincide with the Q-functions of the Bazhanov-Lukyanov-Zamolodchikov opers with the monster potential, which are related to the quantum KdV flows according to the ODE/IM correspondence. We give supporting evidence for this conjecture.

Giulio Ruzza (University of Lisbon, Lisboa, PT)

Title: *Determinantal Point Processes and Integrable PDEs*

Abstract: I will explain how multiplicative statistics of universal determinantal point processes are connected to integrable PDEs. For instance, all cylindrical KdV solutions can be constructed from the Airy determinantal point process as multiplicative expectations, or, more generally, as Jánossy densities. The methods of integrable systems, particularly Riemann-Hilbert techniques, can be applied to asymptotic problems in probability theory. Specifically, I will outline an application to the tail asymptotics of cylindrical KdV and of narrow-wedge KPZ solutions. This talk is based on joint works with Mattia Cafasso, Christophe Charlier, Tom Claeys, Gabriel Glesner, and Sofia Tarricone.

Rosa Sena Dias (IST, Lisboa, PT)

Title: *Conformally Kähler, Einstein Metrics from ODEs*

Abstract: Einstein metrics have played a fundamental role in geometry. One of the main sources of examples of Einstein metrics has been Kähler geometry. Yet we know that several Kähler manifolds do not carry Kähler-Einstein metrics. In the 80's, Derdziński obtained a local construction for 4d conformally Kähler, Einstein metrics using extremal Kähler metrics. Together with G. Oliveira, we set out to use Derdziński's methods on a class of extremal Kähler metrics arising from an ansatz due to Calabi. In this talk, I shall report on our findings. I will start by giving background on Einstein metrics, Kähler geometry, and Derdziński's construction. Calabi's metrics arise from an ODE, and I will explain how this plays a role.

Claude Sabbah (Ecole Polytechnique, Palaiseau, FR)

Title: *On coalescing singularities in isomonodromic deformations*

Abstract: A result of Cotti, Dubrovin and Guzzetti asserts (roughly) that in an isomonodromic deformation of an irregular differential equation of Poincaré rank one and regular semi-simple polar coefficient, the deformation extends to non regular semi-simple polar coefficients only if some specific entries of the Stokes matrix vanish. An analytic approach by Fourier transformation has been given by Guzzetti. We propose a topological explanation of this result by means of the notion of intermediate extension of a local system and topological formulas for the Stokes matrix after Fourier-Laplace transform.

Jacopo Stoppa (SISSA, Trieste, IT)

Title: *TBA*

Abstract: TBA

Dmytro Volin (Uppsala University, Uppsala, SE)

Title: *Geometry of Bethe algebra from ODE/IM*

Abstract: We use a linear problem of ODE/IM correspondence to get an insight on a geometrical way to describe the algebra of conserved charges in integrable models. On some explicit examples, we demonstrate that the proposed point of view yields a more efficient way to compute the spectrum if compared to conventional Bethe equations or QQ-relations.

Xiaomeng Xu (School of Mathematical Sciences, Beijing, CN)

Title: *Stokes matrices, isomonodromy deformation and quantization*

Abstract: This talk focuses on several aspects of the Dubrovin connection. It first derives the explicit formula for its Stokes matrices by studying the long-time behavior of the associated isomonodromy equation. Based on this formula, it then examines the relationship between the WKB approximation of the Stokes matrices and the periods on the underlying spectral curves. Finally, if time allows, it will introduce the quantum Stokes matrices of a quantum ordinary differential system with a k-th order pole, which give rise to a quantization of the irregular Riemann-Hilbert maps at a k-th order pole, as a homomorphism between two associative algebras that quantize the Poisson structures on the de Rham and Betti spaces, respectively.